Short Papers

Microwave Permittivity Measurement Using a Multipoint Technique

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Abstract—**A multipoint technique for complex permittivity measurements is described. This method is based on experimental determination and fitting of the field pattern existing in the sample placed inside a short-circuited slotted waveguide and verified with experimental results.**

I. INTRODUCTION

The usual techniques for measuring complex electrical permittivities at microwave frequencies are of the point type, in that they yield information of the propagation constant in the sample from a minimum number of experimental parameters. Thus, for example, the Roberts–von Hippel method [1], one of the most widely used at these frequencies, in which the measurement of the standing-wave ratio, the location of a minimum of the field, and the wavelength in the medium outside the sample, permits the knowledge of the propagation constant in the sample and, from this, the electrical permittivity of the material.

There are also precision multipoint techniques [2] in which a large number of data points are averaged by means of curves that are selected by analytic curve fitting techniques, though they are best suited to liquids, in which is possible the continuous variation of some experimental parameter, or otherwise by means of sliding terminations that usually introduce additional errors.

The method described here is of the precision type, and is based on the experimental determination of the field pattern in the sample, which is arranged as specified later, and on the least square fitting of the said pattern in order to obtain the wave propagation constant in the material. This method is not only applicable to liquids but also to solids, and this constitutes the aim of our measurements.

II. FUNDAMENTALS OF THE METHOD

The experimental arrangement is similar to that used in the Roberts–von Hippel method, as shown in Fig. 1, it being thus possible to obtain experimentally the existing field pattern just outside the sample.

The propagating mode in the waveguide is the TE_{10}, in which the electric field is in the y direction, being perpendicular to the air-dielectric interface in the upper part of the sample [Fig. 1(b)]. Then the detected field $E_0(z)$ is related to that existing inside the sample $E_i(z)$ by the equation

$$E_0(z) = \epsilon_r E_i(z) \tag{1}$$

where ϵ_r is the relative permittivity of the material. This field $E_0(z)$ must be detected just next to the sample because it is rapidly attenuated by placing the electrical probe away from the slot.

If β is the complex propagation constant in the dielectric and taking the short circuit as the origin of the z axis, the analytic expression of the field, discarding for the moment the possible generation of new modes, will be

$$| E(z_i) | = A \mid \exp (j\beta z_i) - \exp (-j\beta z_i) \mid \tag{2}$$

that is to say, a superposition of the waves incident and reflected at the short circuit; however, the experimental values are usually proportional to the square of this magnitude because of the response law of the detector. In (2), z_i represents the distances from the observation points to the short circuit, and A is a constant which is

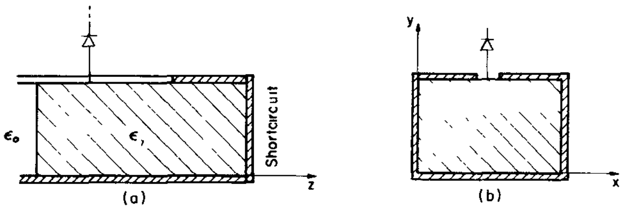

Fig. 1. Measurement cell.

eliminated in the numerical process described later. As previously mentioned, this numerical process results in the complex propagation constant in the material, which is related to its relative dielectric constant by [1]

$$\epsilon_r = \frac{\beta^2 + (2\pi/\lambda_c)^2}{(2\pi/\lambda_0)^2 + (2\pi/\lambda_c)^2} \tag{3}$$

where λ_c is the cutoff wavelength in the guide and λ_0 the wavelength in the medium preceding the sample.

For greater speed in the numerical fitting process, an approximate knowledge of the value of β is necessary:

$$\beta_{ap} = \beta_{ap}' - j\beta_{ap}''. \tag{4}$$

This value may be obtained by means of the position of two consecutive minima, z^1 and z^2, and from the field value in one of them E^1 when the field values have been normalized relative to the maximum of the pattern. Then, it may be easily obtained from (2) that, approximately,

$$\beta_{ap}' = \pi/(z^1 - z^2)$$
$$\beta_{ap}'' = E^1/z^1. \tag{5}$$

III. METHODS OF COMPUTATION

Applying the least square fitting technique, the numerical process is based on making a sweep of the β values around the approximate one β_{ap}, the correct value of β being that which gives the minimum of the variance. The width of the sweep has to be narrow enough to prevent the numerical process from introducing an appreciable error; for this reason, the scanning has been made in two successive iterations, initially with intervals of $\pm$ 5 and $\pm$ 20 percent around β_{ap}' and β_{ap}'', respectively, sampling 100 points in such intervals, and then repeating the same procedure but with division of the intervals by 100 around the previously found optimum β. This procedure results in a final precision of 0.001 percent in β' and of 0.004 percent in β'', surely greater than the experimental one, estimated, as is later seen, at 0.05 percent and a few percent, respectively.

A method of linearization [3] has also been tested as a fitting technique. This procedure consists of developing the function f:

$$f(z,\beta',\beta'') = A \mid \exp (j\beta z) - \exp (-j\beta z) \mid$$
$$= A[2(\cosh (2\beta'' z) - \cos (2\beta' z))]^{1/2} \tag{6}$$

which must fit the experimental data Y_i in a Taylor series rounded off in the first powers around β_0' and β_0'', the supposed solutions of our problem which minimize the variance:

$$\text{var} = \sum_i (Y_i - f(z_i,\beta',\beta''))^2. \tag{7}$$

The transcendental equations which characterize the least square method:

$$\frac{\partial \text{ var}}{\partial \beta'} = \frac{\partial \text{ var}}{\partial \beta''} = 0 \tag{8}$$

lead, in this case, to a system of linear equations. By giving approximate initial values to β' and β'' it is possible to establish an

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iterative correction process from the solution of the linear system. This process converges in our case due to the fact that we possess a sufficiently approximated initial value of the propagation constant.

However, the results obtained led us to reject this method in favor of the one previously described, because, though the time spent on the fitting process is less, its convergence possesses an oscillating character with appreciable amplitudes which prevent us from obtaining sufficiently accurate results.

An alternative procedure which we have tried was to apply the Newton–Raphson technique [4], in order to solve the transcendental equations involved. The resulting equations were similar to those in the linearization method but included terms with second derivatives in the function $f(z,\beta',\beta'')$, while in the linearization method the development is stopped at the first derivatives. However, the results obtained with both techniques are not very different, although, including the second derivatives, the oscillating amplitudes in the convergence process were slightly reduced.

IV. RESULTS

A. *Slotted Section Calibration*

The proposed method is very useful for carrying out a calibration of the slotted section being used as a measurement cell, by fitting the field pattern which exists in the cell in the absence of the sample.

Working at a frequency of 8.9988 GHz with a Hewlett-Packard X810 B slotted guide, the resulting propagation constant was

$$\beta = (1.2893, -17.1 \times 10^{-4})\ \text{cm}^{-1}. \tag{9}$$

Using the value obtained for the real part and also $a = 2.285$ cm for the width of the guide, it may be shown that the resulting precision in the determination of the wavelength is 0.05 percent and, consequently, according to (3), the resulting precision in the determination of the real part of the permittivity will be in the order of 0.1 percent. On the other hand, the waveguide losses (and the sensitivity of the detection system) produce a loss angle:

$$\tan\delta = 12 \times 10^{-4} \tag{10}$$

a value which must be subtracted from the actual results in the measurement of materials, and which, at the same time, constitutes a limit in the sensitivity of the proposed method with respect to loss measurements.

B. *Dielectric Permittivity Measurements*

In the first place we present the results obtained from measurements carried out over samples of various commercial plastics. Both methods, the proposed one and that of Roberts–von Hippel (in the following, methods 1 and 2, respectively) were applied. The results are shown in Table I.

In Table I, the symbols M, B, and E correspond to Metacrilate (Perspex), Bakelite, and Ebonite, respectively, while the numbers accompanying these symbols show the sample lengths in centimeters. In samples of such a length, 10 and 11 cm, and, at the working frequency 9.0 GHz, the problem of multiple roots of the transcendental equation involved in method 2 is not easily overcome by means of the technique of jointly processing the experimental data obtained in two samples of different length. This is due to the fact that the extreme length of the samples results in large values of the zeros of the transcendental equation, and, therefore, the relative spacing between those zeros is very small. As this problem does not appear in the proposed method, the values obtained from it for the propagation constant have been used in order to initiate the resolution of the transcendental equation in method 2. A simplified program of that described in [5] has been developed for these calculations.

The differences between the values of the real part of the permittivity ϵ' for the same material are all less than 0.6 percent. With respect to the losses, the results obtained from one method in different samples differ up to 12 percent in the case of Ebonite, whose small losses border upon the limit of sensitivity of the measurement methods. However, in the case of greater losses, the differences are less than 2 percent. The results obtained for the losses by the two methods of measurement carried out over the same sample agree in every case except for Bakelite, which presented the greatest permittivity and losses of those materials measured. These differences can be explained by the perturbation caused by the nonshorted end of the sample [6]. Whatever irregularity exists in the cutting of this end will generate transmission modes higher than the fundamental one in the guide, which, although being evanescent, will receive a certain amount of power, resulting in an apparent increase in the measured losses. This problem will always be present in the Roberts–von Hippel method due to the fact that the measurements are carried out in the medium preceding the sample. However, with the proposed method, we can eliminate the perturbation caused by the generated new modes simply by making the measurements at points far enough from the nonshorted end of the sample, as shown in the data presented later.

TABLE I
EXPERIMENTAL VALUES OF COMPLEX PERMITTIVITY FOR KNOWN MATERIALS

Sample	Method 1		Method 2	
	ϵ'	$\tan(\delta)\times10^4$	ϵ''	$\tan(\delta)\times10^4$
M 10	2.611	145	2.618	142
M 11	2.613	142	2.627	142
B 10	3.361	322	3.361	374
B 11	3.356	334	3.352	373
E 10	2.636	28	2.640	29
E 11	2.620	32	2.633	30

If there is any evidence of the existence of propagating modes, other than the fundamental, there are two possible solutions. One may take them into account by modifying (2), or otherwise by making a transition to another waveguide of reduced dimensions in order to convert the propagating new modes into evanescent modes. We have observed that these problems and others, such as slot leakage, become increasingly important, at 9 GHz, with relative permittivities in the order of 10.

Another problem that may also be present is the existence of a gap between the sample and the broad wall of the slotted line, our experience being that it is better to introduce a known little gap and then to correct the results [7].

The results of the measurements carried out over a sample of Metacrilate, 11 cm long, at a frequency of 9.0 Ghz, are shown in Table II. These results were obtained by fitting various numbers of field values in different zones of the sample.

Taking as the most probable value for the permittivity that obtained by fitting 40 field values located in zones far enough from the nonshorted end of the sample, shown in line 12 of Table II, we find that the value of ϵ' which most differs from that stated is that obtained by fitting a few points (10) in places near to the end of the sample (line 7), and, even in this case, the difference is less than 0.3 percent, being less than 0.1 percent in the remaining cases, a precision which had already been anticipated for the proposed measurement method.

With respect to the losses, in the two cases corresponding to lines 3 and 7, in which a few values around the top of the field pattern are fitted, the results differ more than 20 percent relative to the initial approximated values obtained by means of (5), a deviation which

TABLE II
RESULTS OBTAINED FOR ONE SAMPLE OF METACRILATE 11 CM LONG

N[a]	Location[b]	ϵ'	$\tan(\delta)\times10^4$
5	8.0- 8.25	2.619	119
10	8.0- 8.5	2.6176	111
10	8.5- 9.0	2.6177	93
10	9.0- 9.5	2.6193	120
10	9.5-10.0	2.6205	121
10	10.0-10.5	2.6203	120
10	10.5-11.0	2.6261	93
20	8.0- 9.0	2.6176	114
20	9.0-10.0	2.6196	119
20	10.0-11.0	2.6176	132
30	8.0- 9.5	2.6186	116
40	8.0-10.0	2.6189	116

[a] N is the number of field points in the fitting process.
[b] Range of distances (in centimeters) from the short circuit.

constitutes the limit provided in the numerical program. This situation is a consequence of the accuracy that may be obtained from the shape of the maxima in the presence of losses.

In the case of line 10, the losses differ by 13 percent, due the aforestated problem caused by the nonshorted end of the sample; in fact, the apparent losses obtained are greater than the correct one. In the remaining cases the differences are less than 4 percent.

V. CONCLUSIONS

A method for measuring the complex electrical permittivity at microwave frequencies has been developed. It consists of the experimental determination and subsequent fitting of the field pattern existing in a sample placed inside a slotted waveguide closed by a short circuit.

The estimated precision of this method is about 0.1 percent for the real part of the permittivity and a few percent for the losses.

Owing to the fact that the oscillator in the experimental setup works under constant load conditions, it is not necessary to achieve a good isolation between the oscillator and the load, this being a problem which is present in the greatest part of waveguide measurement methods.

In comparison with the Roberts–von Hippel method, the proposed one offers the advantage that the possibility of false results is eliminated. On the other hand, it has been found that any irregularity existing in the nonshorted end of the sample will generate new modes which, even being evanescent, will produce an apparent increase in the loses of the material, if the measurements are carried out in the medium preceding the sample, as they are in the previously mentioned method. This problem may be eliminated with the proposed method, carrying out the measurements far enough from the end of the sample, at least when measuring materials with sufficiently low permittivities, to prevent new propagating modes.

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A Lumped-Element Circulator without Crossovers

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***Abstract*—It is proposed to construct a simple "crossoverless" lumped-element circulator, which can be made without sophisticated thin-film processing. The circulator can be described by a "delta connected" equivalent circuit. A simple capacitor arrangement can be used to influence the three eigenvalue phases of the circulator independently, thus permitting this circulator to be maximized systematically. A set of computer-generated eigenvalues gives insight into the behavior of the device under varying operating conditions. Preliminary measurements using a very simple pattern on a 0.650-in-diam ferrite substrate show a 20-dB bandwidth of 10 percent and an insertion loss < 1 dB (0.3 dB/min) at *L* band.**

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INTRODUCTION

The construction of lumped-element circulators at higher frequencies [1]–[3] was made possible by using thin-film crossover techniques which, together with a new approach to broad banding, lead to a completely symmetrical broad-band lumped-element circulator. In order to eliminate processing problems at even higher frequencies, some studies were made to construct a small circulator which does not necessitate any complicated processing. Some preliminary L band results show that it is possible to build such a circulator. The theory proposed seems to be substantiated by eigenvalue measurements on a computerized network analyzer.

There exists a certain similarity with the ring circulator [3], [4]. However, the ring circulator uses three delta connected nonreciprocal phase shifters and the devices realized are significantly larger and more complicated than the present device.

THEORY

A lumped-element circulator with its Y-connected equivalent circuit shown schematically in Fig. 1 can be represented by an impedance matrix of the general form

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad \text{or} \quad [V] = [Z_1][I]$$

or in admittance form:

$$[I] = [Z_1]^{-1}[V], \qquad \text{for } [Z_1]^{-1} = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix} \tag{1}$$

where, in the lossless case,

$$\operatorname{Re}(\alpha) = 0$$
$$\beta = -\gamma^* \tag{2}$$

γ^* being the complex conjugate of γ.

Y and Z matrices have been chosen for the analysis because they can be easily related to the equivalent circuit.

In the special case of Fig. 1,

$$\alpha = \omega G(j\mu)$$
$$\beta = \omega G\left(-\frac{j}{2}\mu - \frac{k}{2}\sqrt{3}\right)$$
$$\gamma = \omega G\left(-\frac{j}{2}\mu + \frac{k}{2}\sqrt{3}\right) \tag{3}$$

where G is a geometry factor [1], [6] which determines the inductance used in the equivalent circuits, and μ and k are the elements of the Polder tensor:

$$\mu_{ik} = \mu_0 \begin{bmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

In the case where losses are taken into account, μ and k become complex numbers which depend on frequency, applied magnetic field, and material parameters. It can be shown [6] that the matrix equation (1), which is valid for the Y-connected three port, is also valid for a Δ-connected one if the notation in Fig. 2 is used, i.e., Y "line" currents (I) become Δ "branch" currents (I). Since we are interested in terminal quantities, the matrix equation

$$[I] = [Z_1]^{-1}[V] \tag{5}$$

has to be transformed into

$$[i] = [Y_1][u] \tag{6}$$

using the notation in Fig. 2, where [6]